QUANTUM PHYSICS I - Nov. 3, 2016

On the first sheet write your name, address and student number. Write your name on **all** other sheets. The total number of points is 90.

PROBLEM 1: UNCERTAINTY PRINCIPLE (points: 5+5+5)

a) What is the definition of the variance (or standard deviation) of an operator with respect to the wavefunction $\psi(x)$?

b) Apply the uncertainty principle to the operators L_x and L_y. Which inequality does this give rise to?

c) For a three-dimensional state with spherical harmonics Y_l^m with $(l, m) = (1, 0)$, can one know both the L_x and L_y values simultaneously with arbitrary high precision? Briefly explain your answer.

PROBLEM 2: ABSTRACT OPERATORS (points: 5+5)

Consider operators on a finite-dimensional vector space in this question.

a) What is the definition of a Hermitian operator, and which property holds for the eigenvalues of Hermitian operators?

b) Is the spectrum of eigenvalues discrete, continuous or can it be both? Briefly explain your answer.

PROBLEM 3: SPIN ADDITION (points: 5+5+10)

a) In the case of two electrons, how do the different spin configurations (with respect to the separate S_z^1 and S_z^2 operators) combine into spin configurations of the full system? Write down the possible quantum numbers S and M of the total system, and give these states in terms of the separate electron spins. Indicate how the ladder operators

$$S_\pm = S_x \pm iS_y \,, \qquad (1)$$

relate the different states.

b) In the case of three electrons, how many spin configurations (with respect to S_z^1, S_z^2 and S_z^3) are there? How do these combine into spin configurations of the full system? Write down the possible quantum numbers S and M of the total system, and indicate how the ladder operators relate the different states. You do not have to give these states in terms of the separate electron spins.

c) Derive the states of the full system in terms of the separate electron spins for the case of the longest ladder(s), i.e. with highest value of S, of the previous question.

PROBLEM 4: HARMONIC OSCILLATOR (points: 5+5+5+10+10+10)

The Hamiltonian for the harmonic oscillator (in one dimension) is given by

$$H = \frac{p^2}{2m} + \frac{m\omega^2 x^2}{2}\,. \tag{2}$$

It will be convenient to define ladder operators

$$a_\pm = \frac{1}{2\hbar m\omega}(\mp ip + m\omega x)\,. \tag{3}$$

a) Write the Hamiltonian in terms of ladder operators.

b) What is the energy of the ground state $\psi_0(x)$? Is this different from the classical answer? Explain your answer.

c) What is the form of the energy spectrum, and how can one construct the corresponding energy states (you don't have to do this explicitly)?

d) The first excited state is given by

$$\psi_1(x) = A_1 x e^{-\frac{m\omega}{2\hbar}x^2}\,. \tag{4}$$

What is the correct normalization of the first excited state?

e) What are the expectation values of the operators kinetic energy T and the kinetic energy V for a particle in the first excited state?

f) Derive the form of the second excited state, up to an arbitrary normalization.